ANOMALOUS EFFECT OF SURFACE TEMPERATURE ON
STABILITY OF A LAMINAR GAS BOUNDARY LAYER

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It is well known that the stability of a gas boundary layer is increased by cooling the surface, while heating of the surface decreases stability. The earliest studies considered cases of constant surface temperature. In [3] a study was performed in the presence of a temperature gradient along the surface. It was shown that nonuniformity of the surface temperature distribution with retention of the total thermal flux at the same value as for constant temperature can have a significant effect on the boundary layer stability characteristics and may lead to significant displacement of stability loss points both up and down the flow.

In the present study material obtained from a large number of numerical calculations allows study of cases of special interest which radically change concepts of temperature boundary layer stability conditions. Namely, under certain conditions there may even be an increase in critical Reynolds numbers upon heating with a decrease occurring upon cooling. The changes in critical Reynolds number and other stability characteristics may be very significant.

A slight increase in stability of a boundary layer on a plate upon heating of the leading edge was reported in [4].

Formulation of the Problem. We will consider the stability of a planar infrasonic laminar boundary layer with nonuniform surface temperature distribution. As in [3], it will be assumed that the characteristic length for temperature change, which is comparable to the surface size L, is much greater than the wavelength of natural perturbations (Tollmin–Schlichting waves) λ. This makes it possible to use the plane-parallel boundary layer approximation. For the mathematical model we use the system of [3], consisting of the continuity equation, Navier–Stokes equations for the two components of the velocity vector, and the energy equation with corresponding boundary conditions. It is assumed that the gas obeys the equation of state of an ideal gas.

We specify the surface temperature in the form

$$\overline{T}_w = 1 + T_1(a + \xi^n), \tag{1}$$

where $\xi = x/L$ is the dimensionless longitudinal coordinate; the subscripts e and w indicate the surface and the external flow $\overline{T}_w = T_w/T_e$.

In [3] it was assumed that $a = 0$, which corresponds to a surface temperature distribution for which self-similar solutions of the steady-state boundary layer exist, while in the present study we will consider stability of non-self-similar steady-state solutions.

Linearizing the original equations we obtain an equation which describes the development of small perturbations in the boundary layer:

$$\begin{gathered}(u-c)(\varphi''-\alpha^2\psi^2\varphi)-\varphi u''-\frac{\gamma}{i\alpha\,\mathrm{Re}}\left(\frac{\varphi^{\mathrm{IV}}}{\psi}-2\alpha^2\psi\varphi''+\alpha^4\psi^3\varphi\right)=\\ =\frac{1}{i\alpha\,\mathrm{Re}}\left\{\varphi'''\left[2\left(\frac{\gamma}{\psi}\right)'-\chi\frac{\gamma}{\psi}\right]+\varphi''\left[\left(\frac{\gamma'}{\psi}\right)'-2\chi\left(\frac{\gamma}{\psi}\right)'-\frac{\gamma}{\psi}\chi'\right]-\right.\\ \left.-\alpha^2\varphi'(\gamma\psi'+2\psi\gamma')+\alpha^2\psi\varphi\left[\chi(\gamma\chi-\gamma')+\gamma''-\frac{\gamma}{\psi}\psi''\right]\right\}+\chi\varphi'(u-c)-\chi\varphi u'.\end{gathered} \tag{2}$$

with boundary conditions:

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$$\varphi = 0,\ \varphi' = 0\ (\eta = 0),\ \varphi = 0,\ \varphi' = 0\ (\eta = \infty). \tag{3}$$

Here $\varphi(\eta)$ is the amplitude of the flow functions; α, c are the wave number and phase velocity of the perturbation; u, longitudinal velocity component of the main flow; Re, Reynolds number, constructed using the displacement thickness; ν, kinetic viscosity; T, ρ, gas temperature and density; $\psi = T/T_e$ is the current temperature factor, differentiation is performed with respect to the dimensionless coordinate $\eta = \left(\frac{u_e}{x\nu_e}\right)^{1/2} \int\limits_0^y \frac{\rho}{\rho_e} dy$; the prime denoting such differentiation; $\chi = \psi'/\psi$; $\gamma = \mu/\mu_e$.

The operator on the left side of Eq. (3) has the qualitative form of a linear Orr–Sommerfeld operator, the only difference being that here ψ and the relative viscosity appear as factors in a number of terms. The operator on the right of this equation is significantly more cumbersome in form. Analysis reveals that it depends to a significant degree on the thermal flux distribution, both across and along the boundary layer.

Thus, the problem reduces to finding eigenvalues of boundary problem (2), (3). To achieve a solution it is necessary to define the coefficients in Eq. (2) which contain the distributions of velocity, temperature, and viscosity over boundary layer thickness and their derivatives. This means we must know the characteristics of the main flow, which can be found from the equations of the temperature boundary layer. We write these equations in self-similar variables:

$$(Kf'')' + \frac{m+1}{2} ff'' + m(\psi - f'^2) = \Phi_1,$$
$$\frac{1}{\Pr}(K\Theta')' + \frac{m+1}{2} f\Theta' + n(1-\Theta) f' \frac{\xi^n}{a+\xi^n} = \Phi_2. \tag{4}$$

with boundary conditions

$$f = 0,\ f' = 0,\ \Theta = 0\ (\eta = 0),\ f' = 1,\ \Theta = 1\ (\eta = \infty). \tag{5}$$

Here $\Phi_1 = \xi\left(f' \frac{\partial f'}{\partial \xi} - f'' \frac{\partial f}{\partial \xi}\right)$; $\Phi_2 = \xi\left(f' \frac{\partial \Theta}{\partial \xi} - \Theta' \frac{\partial f}{\partial \xi}\right)$; $\Theta = \frac{T - T_w}{T_e - T_w}$; $m = \frac{\xi}{u_e}\frac{du_e}{d\xi}$; $f = \int\limits_0^\eta \frac{u}{u_e} d\eta$; $K = \mu\rho/\mu_e\rho_e$,

Pr is the Prandtl number. The presence of the functions Φ_1 and Φ_2 in Eq. (4) is related to the non-self-similarity of the solutions of the boundary layer equations. In [3] $\Phi_1 = \Phi_2 = 0$.

The study was performed numerically using a computer. To linearize the problem a modernized orthogonality method was used.

Calculation Results. We will present only those results which illustrate the anomalous effect of surface temperature on laminary boundary layer stability characteristics. Figures 1-4 show neutral stability curves and spatial perturbation increment coefficients α_i for fixed values of Re. Figures 1-3 correspond to surface heating, Fig. 4 to cooling. The dashed lines of Figs. 1-4 correspond to an isothermal boundary layer, where the surface temperature is equal to the temperature of the external flow. Lines 1 are for nonuniform heating (cooling) of the surface by the power law of Eq. (1), lines 2, constant surface temperature selected by the condition of equality of net thermal fluxes for the constant temperature and the nonuniform distribution (see [3] for more detail).

We will consider surface heating first. Figure 1 shows calculation data for the following parameters in the surface temperature law: $T_1 = -0.5$, $n = 4$, $a = -1$, i.e., the temperature of the plate. Comparing curves 1 and 2, we see that such a negative temperature gradient leads to an abrupt increase in boundary layer stability, a narrowing of the unstable frequency range, and a decrease in the perturbation spatial increment coefficients. Thus, the minimum critical Re_{min} increases by more than five times, and the maximum value of $\alpha_{i\ max}$ decreases by approximately eight times for the chosen value $\mathrm{Re} = 2 \cdot 10^3$ such an increase in Re_{min} corresponds approximately to a 28-fold increase in Re_{min} calculated using the current length $u_e \ell_{min}/\nu_e$. We note that curve 2 corresponds to $\overline{T}_w = 1.27$.

Even more interesting conclusions follow from a comparison of curves 1 and 3, corresponding to an isothermal boundary layer. It is evident that surface heating leads to an unusual effect – increase in boundary layer stability. Here the unstable frequency ranges narrow and

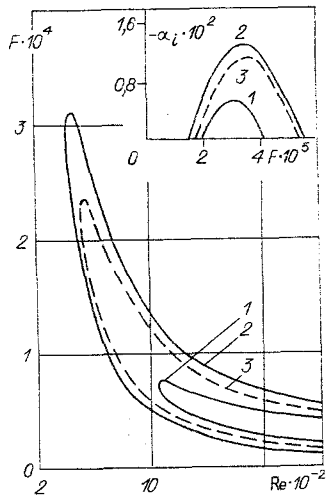

Fig. 1

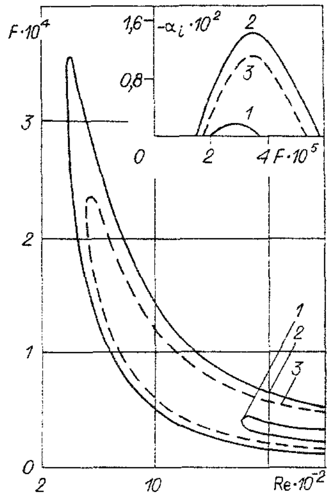

Fig. 2

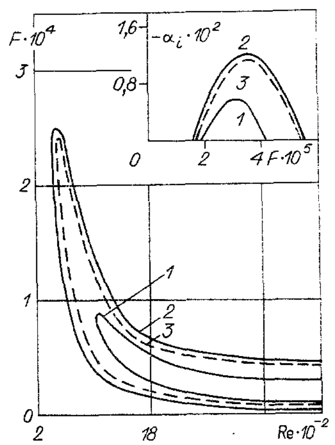

Fig. 3

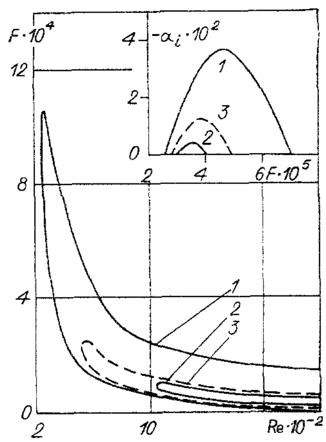

Fig. 4

the coefficients α_i decrease. Thus, the increase obtained in Re_{min} corresponds to an increase in the lengthh ℓ_{min} by approximately an order of magnitude, while $\alpha_{i\ min}$ decreases by a factor of six. Figure 2 shows analogous data for a linear temperature distribution along the entire length of the surface for $T_1 = -0.5$, $n = 1$, $a = -1$, while Fig. 3 is for such a distribution on only the second half of the plate with $T_w = 1 + T_1(a + \xi^n)$, $T_1 = -0.25$, $n = 1$, $a = -1$. All the qualitative features of the effect of surface temperature gradient on stability characteristics remain the same as in the first case (Fig. 1), although they appear somewhat more weakly. For example, for linear decrease in surface temperature along the entire length Re_{min} increases somewhat more than four times. Curve 2 of Fig. 2 corresponds to $\overline{T}_w = 1.14$, while in Fig. 3, $\overline{T}_w = 1.012$.

It is interesting to compare the data presented in Figs. 2 and 3. The temperature gradient along the surface is identical in both cases, but, as has already been noted, in the second case only the second half of the plate is heated, so its temperature head compared to the external flow temperature is twice as small. Meanwhile, comparing the neutral stability curves 1 of both figures, we see that they are very close to each other, Re_{min} in the second case being only 5% larger. If we consider that in the second case the total thermal flux is four times smaller, the data have great significance from the viewpoint of laminarization of the boundary layer. They indicate the inefficiency of heating the initial portion of the surface which lies significantly up the flow from the stability loss point.

This unusual effect of increase in boundary layer stability upon surface heating and in the presence of a negative temperature gradient along the surface has the following physical explanation. On the maximally heated portion of the surface the core of the boundary layer is strongly heated, and its temperature remains higher than the surface temperature down the

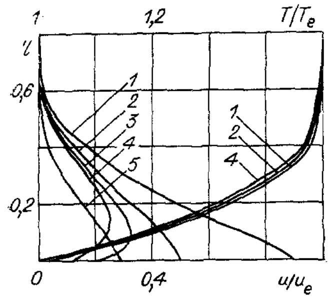

Fig. 5

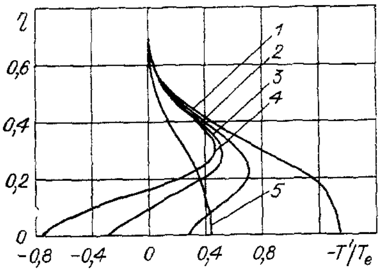

Fig. 6

flow. This means that the thermal flux there is directed toward the surface, despite the fact that the temperature of the latter is higher than that of the external flow. Figures 5, 6 show results of calculating development of a steady-state temperature boundary layer with nonuniform surface heating for linear temperature decrease along the surface. Figure 5 shows dimensionless velocity and temperature profiles in the boundary layer, while Fig. 6 shows a quantity proportional to the change in thermal flux across the boundary layer. Curves 1-4 correspond to $\xi = 0.1$, 0.5, 0.7, 0.9, and curve 5, to uniform surface heating at $\overline{T}_w = 1.14$. It is evident from Fig. 6 that the thermal flux changes direction as early as $\xi = 0.6$. However, the situation being considered should not be identified with "conventional" surface cooling, since in the indicated range the surface temperature is higher than the temperature at the external edge of the boundary layer everywhere.

We will now turn to surface cooling, where the surface temperature increases over length, undergoing the most abrupt change toward the end of the plate, i.e., $T_1 = 0.5$, $n = 4$, $a = -1$. Comparing curves 1 and 2 of Fig. 4 (curve 2 corresponds to a surface temperature $\overline{T}_w = 0.72$), we see that the chosen positive temperature gradient leads to an intense decrease in boundary layer stability, expansion of the unstable frequency range, and increase in the coefficients α_i. Thus, Re_{min} decreases by approximately 4.2 times, while $\alpha_{i\ max}$ increases by almost nine times. Such a reduction in Re_{min} corresponds to a decrease in ℓ_{min} by approximately 18 times.

Now comparing curves 1 with the calculation results for the isothermal case (curves 3), we see that there is a significant decrease in flow stability, expansion of the unstable frequency range, and increase in the increment coefficients. Thus, ℓ_{min} decreases by more than four times, while $\alpha_{i\ max}$ increases approximately three times.

Analogous calculations for other forms of surface temperature distribution during cooling will not be presented here. Their results are of the same qualitative character as those presented above, if the temperature gradient is positive.

The anomalous effect of decrease in stability of a laminar boundary layer on a cooled surface established herein is caused by the fact that because of abrupt reduction in the temperature of the layer core on the maximally cooled portion, down the flow the thermal flux is directed away from the surface, despite the fact that the surface temperature there is less than the external flow temperature.

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